AAE/IS 374 **Brad Barham and John Morrow**
TA: DylanFitz

Problem Set 3:
Conditional Convergence and Examining the Sources of Growth

DUE IN DISCUSSION 23 OCTOBER

Please note that you may turn this problem set in at discussion section on Friday, 17 October. If for some reason you are going to miss sections that day, please turn it in ahead of time. Late problem sets will not be accepted.

1. We have seen that one implication of the Solow model is `convergence'. Convergence means that, all else equal, countries with lower levels of income will grow more rapidly than countries with higher levels of income. In Problem Set 2 you tested whether 'unconditional convergence' held for the data. (This meant that you did not take the 'all else equal' part of the above statement very seriously in the analysis.) After discussing human capital we have now seen that countries with different levels of human capital will have different steady state levels of income. According to the augmented Solow Model, conditional on the level of human capital, countries with lower incomes should grow more quickly.

   The first worksheet of pset3.xls is called 'Conditional Convergence.' This spreadsheet includes data on GDP/worker in 2000, GDP/worker in 1965, average years of schooling in the population over the age of 15 in 1985, and infant mortality in 1985 (deaths per 1000 live births of infants under the age of one).

   a. Group countries into those with low, medium, and high levels of education and for each group of countries draw a scatter plot with 1965 GDP/worker on the x or horizontal axis and the 65-00 annual Growth Rate on the y or vertical axis. Then add in a trend line. Please discuss any patterns apparent in your scatter diagram. Does it appear to support conditional convergence? Does it show divergence, or any other visible pattern? Or is it just a "mess"?

   b. Now group countries into those with low, medium, and high levels of health and repeat the previous exercise. Please discuss any patterns apparent in your scatter diagram. Does it appear to support conditional convergence? Discuss differences and similarities that arise when carrying out this exercise for education versus carrying out this exercise for health.

2. The next part of the course will begin to examine technological change. This problem invites you to begin looking at patterns of technological change in the global economy by

analyzing a total factor productivity measure of technological change.

The 'TFPG' worksheet in pset3.xls provides the data taken from the Penn World Tables and World Bank sources on the following measures for a sample of 54 countries:

- GNP per-worker, 1965 and 1990
- Physical Capital Stock per-worker, 1965 and 1990
- Human Capital per-worker, 1965-1990

Human capital per-worker is defined as the percentage of the working age population that has at least attended some secondary school. (Unfortunately, we have not been able to find an updated capital stock information so that economic performance of more recent years may be analyzed.)

The Weil text (section 7.3) presents a decomposition of growth based on the two factor production function, $Y = AK^{\alpha}L^{1-\alpha}$, showing that $\hat{A} = \hat{y} - \alpha\hat{k}$, where a '^' over a variable means its growth rate (e.g., $\hat{y}$ is the growth rate of *y*).

In discussion sections, Andres will discuss both this two factor decomposition as well as the decomposition for the production function which expresses production as a function of three inputs:

$$Y = AK^{\alpha}S^{\beta}P^{1-\alpha-\beta}$$

where *Y* is output, *A* is the technology multiplier (total factor productivity) term, *K* is physical capital, *S* is human capital and *P* is labor force. Because this is a constant returns to scale production function, we can express per-worker output as:

$$y = Ak^{\alpha}s^{\beta}$$

where as always the lower case letters indicates per-worker figures. (Note, *s* represents per worker human capital in this case.) By taking logs and differentiating, we can decompose the rate of per-capita growth in the economy as:

$$\hat{y} = \hat{A} + \alpha\hat{k} + \beta\hat{s}$$

Simple algebra allows us to rewrite this expression as,

$$(*) \quad \hat{A} = \hat{y} - \alpha\hat{k} - \beta\hat{s}$$

where $\hat{y}, \hat{k}, \hat{s}$ *and* $\hat{A}$ are, respectively, the growth rates of output per-worker, capital per-worker, human capital per-worker, and total factor productivity. The terms α and β are elasticities of output with respect to physical and human capital. For your analysis here, assume that $\alpha = 0.39$ and that $\beta = 0.27$.

a. Using the data for the full sample of countries given in pset3.xls, please calculate the growth rates for GNP per-worker, physical capital per-worker and human capital per-worker over the 1965-1990 period.

b. Using your results from question 1, compare the distributions of the growth rates for per-worker physical capital and per-worker human capital.

    i. Which factor of production has grown fastest over the last 25 years?

    ii. Which factor growth is more variable or exhibits the largest spread?

    Please answer these questions by calculating the mean and the median growth rates for both factors. Also, please calculate the variance and the standard deviation of these rates. Using a histogram to examine the distribution of these growth rates, please indicate which growth rate seems to have varied the most. Please explain your answer carefully and clearly and include your graphs with the material that you turn in. Label your graphs and make clear reference to the graphs and your numerical measures of level and spread in your analysis.

c. Using the expression (*) above, calculate TFPG for the full sample of countries.

d. Calculate for each country the percentage of total growth accounted for by TFPG. (Note, this percentage can be negative if calculated TFPG is negative. A negative number indicates that a country is estimated to have become less good at extracting outputs from its measured inputs of labor, physical and human capital. For such a country to have had its per-capita GDP grow at all, it is necessary for the growth of its inputs to have been strong enough to overwhelm the negative TFPG growth.)

e. Has TFPG been extraordinarily high in East Asia compared to other world regions? To answer this, you will need to examine the distribution of TFPG (and TFPG as a percent of total growth) for East Asian countries versus the distribution for the rest of the world. In answering this question, please examine both the mean and median values of your TFPG measures for both East Asian countries and the rest of the world.