

**The Growth and Development of Nations***Answer Key to Problem Set 1:  
TRADE IN A NORTH-SOUTH MODEL*

This problem set puts forward a “North-South” trade and growth model that allows interactive exploration of the conventional comparative advantage-based theory of trade. Note that we assume that goods are produced with common, constant returns to scale technology that has diminishing returns to increases in any single factor of production (*i.e.*, we use the same technology assumptions that we explored in the Solow growth model).

The problem set hopes to highlight three specific implications of this model. The first is the idea that both North and South benefit from free trade. The second is that growth in the world economy is characterized by a pattern of convergence in which the South grows faster than the North such that Southern income levels eventually catch-up with those of the North. The third, and perhaps most surprising, is the notion that decentralized market regulation of the economy not only sees to it that trade is mutually beneficial, and that growth is convergent, but that it also generates an economic outcome which cannot be improved upon (in a well defined way) given the real structural constraints of technology and resources. Put differently, this third implication is that any non-market regulation of the economy is at best superfluous, and at worst destructive.

1. The simulation model you will analyze below assumes a world of two countries—the North which is assumed to have a relatively abundant supply of physical capital; and the South which has a relatively abundant supply of labor. Both economies can produce a manufactured good (computers, whose production is relatively capital-intensive) and an agricultural good, (bananas, whose production is relatively labor-intensive). Both countries have access to the same technologies which are defined as follows:

$$X_a = L_a^{(1-\alpha)} K_a^\alpha ; \text{ and,}$$

$$X_m = L_m^{(1-\beta)} K_m^\beta .$$

where  $X_a$  is the quantity produced of the agricultural good which can be physically produced from inputs of labor ( $L_a$ ) and capital ( $K_a$ ). Symmetric definitions apply to the manufacturing sector where  $X_m$  is the quantity of computer output, and  $L_m$ , and  $K_m$  are the production inputs. The symbols  $\alpha$  and  $\beta$  simply represent the parameters of the technology and it is assumed that  $\beta > \alpha$  (*i.e.*, the manufactured good is more capital-intensive than the agricultural good). The simulation assumes that  $\beta=0.8$  and that  $\alpha=0.2$ . Please use these same assumptions in answering this question.

- a. Consistent with the simulation model assume that North and South have the following initial endowments of productive factors:

	<i>North</i>	<i>South</i>
<i>Initial Labor Endowment</i>	25	30
<i>Initial Capital Endowment</i>	40	10
<i>Capital-Labor ratio</i>	40/25=1.60	10/30=0.33

Suppose that the North devoted all of its labor and capital to the production of computers. How many computers could it produce in total? How many bananas could it produce if it only produced bananas? What is the ratio of maximal computer to maximal banana production in the North?

If N only produced bananas:  $X_a = 25^{(1-0.2)} * 40^{0.2} = 27.46$  bananas

If N only produced computers:  $X_m = 25^{(1-0.8)} * 40^{0.8} = 36.41$  computers

Max Computer : Max banana = 36.41: 27.46 = 1.33

- b. What is the ratio of maximal computer to maximal banana production in the South?

If S only produced bananas:  $X_a = 30^{(1-0.2)} * 10^{0.2} = 24.08$  bananas

If S only produced computers:  $X_m = 30^{(1-0.8)} * 10^{0.8} = 12.46$  computers

Max Computer : Max banana = 12.46: 24.08 = 0.52

- c. Why are the two ratios that you calculated in parts (a) and (b) different? Explain why the North have a comparative advantage in computers, and why the South has a comparative advantage in bananas.

The ratios are different due to the differences in factor intensity and factor endowment of each country. Computers are relatively capital-intensive and if S tries to produce very many computers, they quickly run out of the needed capital. S thus ends up producing computers with the wrong factor mix (lots of labor, and a little capital spread out amongst the many laborers). In the end, S can get relatively few computers even if it devotes its entire stock of labor and computers to the effort.

Bananas, on the other hand, are relatively labor intensive. When N tries to produce a lot of bananas, it quickly runs out of labor that is needed to produce bananas (as N has lots of capital, but relatively little labor).

It is exactly the disadvantage of S in computer production, and disadvantage of N in banana production that creates the different comparative advantages of the two countries. N has comparative advantage in C production and S has comparative advantage in B production.

- d. Now let's approximate the marginal productivity of capital in both economies when they engage in maximal computer production. To do this, calculate how many *additional* computers the north can maximally produce if we give them one more unit of capital (so that they are producing computers with 25 units of labor and 41 units of capital). Calculate the same thing for the South (when they are

given one extra unit of capital for a total of 11 units).

N production level with one additional unit of capital:

$$X_m = 25^{(0.2)} * 41^{0.8} = 37.14, \quad x_m - 36.41 = 0.73 \text{ additional computers}$$

S production level with one additional unit of capital:

$$X_m = 30^{(0.2)} * 11^{0.8} = 13.44, \quad x_m - 12.46 = 0.99 \text{ additional computers}$$

- e. What do you think this difference in the marginal productivity of additional capital means for the ability of South to converge with North?

The implication here is that the output bang for the additional savings buck will be larger in S than in N. It is this difference in marginal productivity of capital between capital scarce and capital abundant countries that underlies the idea that S will eventually converge with N (assuming that S has a savings rate no less than that in N).

**2. Autarchy and Differential Scarcity in North and South**  
**run autarchy.gcg <<enter>>**

**Table 1a: Economic Scarcity in the South**

Initial Level of Computer Production which you choose	Number of Units of Bananas given up when four more computers are produced	Relative Price of Computers (Number of bananas given up per computer at this margin of computer production)
2	5	1.25
4	6	1.50
6	9	2.25
8	12	3.00

**Table 1b: Economic Scarcity in the North**

Initial Level of Computer Production which you choose	Number of Units of Bananas given up when four more computers are produced	Relative Price of Computers (Number of bananas given up per computer at this margin of computer production)
8	2	0.50
16	2.4	0.60
20	3.2	0.80
28	4	1.00

b.

**Table 2: Autarchy Equilibrium**

	Production		Consumption		Equilibrium Prices				Income		Consumer Utility
	<i>Ban.</i>	<i>Comp.</i>	<i>Ban.</i>	<i>Comp.</i>	<i>Ban.</i>	<i>Comp.</i>	<i>Lab.</i>	<i>Cap.</i>	<i>Total</i>	<i>Per-Capita</i>	
South	14.6	7.6	14.6	7.6	1	1.93	0.49	1.46	29.2	0.97	10.5
North	16.7	22.1	16.7	22.1	1	0.75	0.66	0.41	33.3	1.33	19.2

c. Prices:

1. In what sense are the numbers you calculated for the third column of Tables 1a and 1b "relative prices"?

Actual prices don't really matter, only relative prices matter. (You could say a candy bar costs \$1 or 100 cents, all that matters is how much it costs relative to your salary and how much it costs relative to everything else you have to buy.) The relative price is how many bananas it would take to buy a computer, and that is what the relative price is, and that is all that matters.

2. Focusing on the numbers for the south, what happens to the relative price of computers as the level of computer production increases?

As computer production goes up the relative price of computers goes up.

3. Intuitively, explain why this happens?

As computer production goes up, more labor is working on building computers and there are diminishing returns. Labor is more efficient working on banana production, but as the South builds more computers and takes people away from the banana sector and moves them into the computer sector, the people are less efficient, and so the more computers a country makes the more costly it is in terms of banana production they give up.

d. Comparing Prices:

1. How do relative prices for computers compare between North and South?

The relative price of computers is higher in the South at any production level.

2. Why are these prices different at the same level of production?

These prices are different because the South is labor intensive while the North is capital intensive. Since the South has relatively more labor, the South will always be better at producing bananas and worse at producing computers than the North.

3. Assuming identical consumer preferences between North and South (as in this model), what would you expect production patterns between the two countries to look like in autarchy in terms of the relative and absolute numbers of computers produced and consumed (given that the North is relatively well endowed with capital as assumed here)?

I would expect the South to produce and consume relatively more bananas and the North to produce and consume relatively more computers, since they should both produce relatively more of the good whose production is intensive in the factor they have more of (and they can't trade so they must consume what they

produce). We cannot be sure of the absolute numbers of bananas and computers produced. That would depend on the size of the economies and the exact production functions. The North could produce absolutely more bananas and computers or absolutely less than the South.

e. **Production and Prices:**

1. Do the autarchy equilibrium production patterns bear out your expectations?

The equilibrium production patterns bear out my expectations in terms of relative production and consumption. The North produces more computers than bananas, while the South produces less computers than bananas. Thus, the North produces relatively more computers. In this case, the North happened to produce absolutely more computers than the South (and absolutely more bananas as well).

2. How do the equilibrium prices for computers compare to the economic scarcity prices you calculated earlier for the two economies?

They are close, considering that my numbers were calculated by eyeballing a graph (over a difference of four computers) while the ones that Gauss gave me were calculated exactly by the computer itself for a marginal change in computer production. I estimated a relative price of computers of 1.5 in the South at 4 computers and 3.0 at 8 computers, while Gauss told me the relative price was 1.93 at production of 7.6 computers. I estimated a relative price of computers of 0.6 in the North at 16 computers and 1.0 at 28 computers, while Gauss told me the relative price was 0.75 at production of 22.1 computers.

3. Why aren't more computers produced and consumed in the south given that southern consumers have the same love of computers as northerners?

The people in the South don't produce more computers because each computer they produce gets more and more costly in terms of the bananas they must give up to produce each extra computer. The cost of computers discourages southern computers from buying many computers and they substitute into bananas. Since the South is relatively abundant in labor but has relatively scarce capital, their comparative advantage is in growing bananas. They don't have enough capital to make it worthwhile to build more computers.

f. **Consumers:**

1. If you were a southern consumer, would you be interested in access to the international market?

Definitely!

2. Why or why not?

If I were a southern consumer I could buy a ton of bananas for \$1 (their price in the South) and then I could give that ton of bananas to someone in the North in exchange for a computer. In the north a computer only costs 0.75 tons of bananas. I could tell the person I was buying it from to keep the change, and just give me one computer in exchange for a dollar's worth of bananas. In the south that same computer would have cost \$1.93, something I wouldn't have had enough money for if I only had one ton of bananas.

3. If you were a southern worker, would trade liberalization appear promising?

If I were a southern worker, trade liberalization would appear very promising. I would know that before trade my wage was low because there was a lot of labor in my country. Once we open ourselves up to trade my wage should go up since there are fewer laborers in the North and since their wage is higher. The Factor Price Equalization theorem says returns to factors should be equalized across countries, and as a southern worker that sounds good to me since labor earns more in the North.

4. What if you were a southern capitalist (the owner of capital)?

I wouldn't be so keen on trade liberalization if I were a southern capitalist. The same argument used above could be used to show that the return to capital will go down once we start trading. I earn a lot before trade since the capital I own is quite rare. Once we start trading with the North my capital becomes less special and so I earn less of a return on it.

**3. Static Gains from Trade**

a. *Trading from the autarchic production pattern*

This first part of the problem studies what happens when Southern producers do not adjust their production patterns to international trading opportunities, but Southern consumers are allowed to buy foreign goods at a set of given world prices. You get to do the shopping for Southern Consumers (!) by deciding how many computers you would like to buy. The computer will automatically adjust to your choices by making sure that enough Southern Products are exported so that you have the income (and foreign exchange) needed to pay for your computer shopping.

**run shop.gcg <enter>**

**Table 3: Autarchic Production Patterns for the South:**

Computers: 7.6  
 Bananas: 14.6  
 World Prices:  
 Bananas: 1  
 Computers: 0.75

Computer Consumption <Your choice>	Exports < Implied by production patterns and trade balance >	Imports < Implied by production patterns and trade balance >	Banana Consumption	Consumer Utility
4	3.55 computers	3.76 bananas	18.36	8.57
8	0.47 bananas	0.45 computers	14.13	10.63
12	4.71 bananas	4.45 computers	9.89	12.10
16	8.94 bananas	8.45 computers	5.66	9.51
20	13.18 bananas	12.45 computers	1.42	5.33

*b. Free Trade Equilibrium*

run free.gcg <enter>

**Table 4: Free Trade Equilibrium**

	South	North
Equilibrium Prices		
Bananas	1.00	1.00
Computers	1.06	1.06
Labor	0.59	0.59
Capital	0.65	0.65
Bananas		
Production	21.61	11.10
Exports	9.42	0
Imports	0	9.42
Consumption	12.19	20.52
Computers		
Production	2.62	28.28
Exports	0	8.89
Imports	8.89	0
Consumption	11.52	19.38
Consumer Utility	11.85	19.94

c. *Analysis:*

1. What are the most striking changes in the free trade versus the autarchy equilibria you examined in question 1 above?

It is striking that now the countries consume much more equal quantities of computers and bananas. In the North they consume around 20 of each and in the South around 12 of each. Before the South consumed much more bananas and the North consumed much more computers.

2. What results most surprised you?

How much capital in the South was hurt. They used to earn 1.46 and now only 0.65. (Of course there is no right answer to this question and the question above.

3. Are the gains from free trading “large”?

Utility in the South went up from 10.5 to 11.85 comparing autarchy to free trade. Utility in the North went from 19.2 to 19.94. These gains from trade seem smaller than I might have expected them to be. Free trade leads to a 13% increase in utility in the South and 4% increase in the North. I guess whether that is large or not is relative.

*d. Pareto Optimality of Free Trade Equilibrium*

**run pareto.gcg <enter>**

**Table 5: Plan versus Market in the South**

Southern Consumer Utility under Autarchy: 10.5

Southern Consumer Utility under Free Trade: 11.85

Computer Production <Your choice>	Implied Banana Production	Computer Consumption <Your choice>	Implied Banana Consumption	Consumer Utility
1.46	22.78	11.00	12.68	11.81
2.54	21.70	11.00	12.74	11.84
2.54	21.70	12.00	11.68	11.84
3.44	20.71	12.00	11.65	11.82
7.26	15.16	12.00	10.14	11.03

*e. Results:*

1. Were you as central planner able to beat the market?

I was not able to beat the market. I did get close in my last choice, but that is a bit unfair, since I knew the result that the market would have chosen so I could make my choices better based on that.

2. Are you a free trader now (explain your answer)?

There is no right answer for this. This shows that trade is good for the South and the North. Of course the model is based on some simplified assumptions (similar access to technology, constant returns to scale, etc.) if some of these didn't hold the results as to whether free trade was good or bad might differ.